Basic Algorithms for Digital Image Analysis

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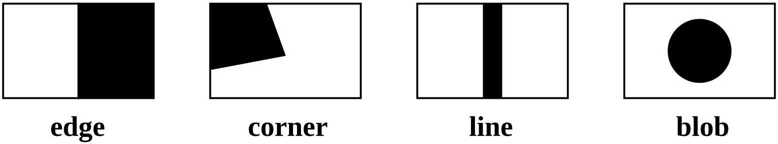
Faculty of Informatics

Edge detection

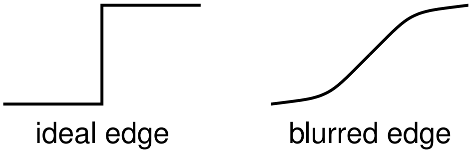
Basic image features



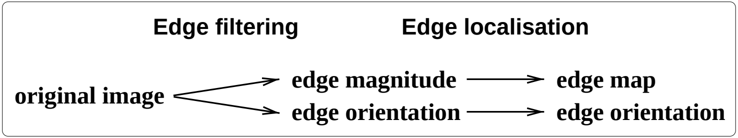
- **Edge**: drastic change of intensity across object contour
  $\Rightarrow$ this lecture
- **Corner**: sharp turn of contour
  $\Rightarrow$ next lecture
- **Line**: narrow, elongated region of approx. constant width and intensity
- **Blob**: compact image region of approx. constant intensity

Image edges

- **Image edges** do not necessarily coincide with **physical edges**
  - image edges are intensity discontinuities
  - physical edges are physical surface discontinuities
  - $\Rightarrow$ edges of *shadows* are not surface discontinuities
- Importance of intensity edges
  - human eye detects them 'in hardware', at initial level of visual processing
  - we see differences not absolute values $\Rightarrow$ adaptivity



Steps of edge detection



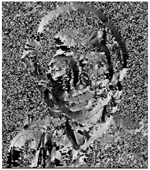
- **Edge filter** responds to edges and yields
  - *edge magnitude*: strength of edge, local contrast
  - *edge orientation*: circular data, mod π
- **Edge localisation** (post-processing)
  - removes noisy edges
  - removes ‘phantom’ edges, obtains thin contours
  - obtains *edge map*: binary edge image

$\Rightarrow$ **Noise smoothing** may be applied before edge filtering

Example of edge detection by 3 $\times$ 3 Prewitt operator


original image


edge magnitude


edge orientation


edge map

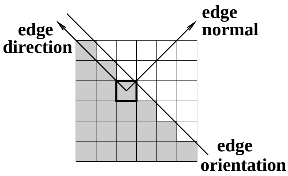
- Edge orientation is circular data: flips at 0 and π


upper line


lower line

Intensity profiles of original image along two lines

Edge normal, edge direction and edge orientation



- **Edge normal**: Direction of maximum intensity variation at edge point
  - unit vector perpendicular to edge
- **Edge direction**: Direction tangent to contour
  - unit vector parallel to edge
  - convention needed for unambiguous definition
  - $\Rightarrow$ 'dark on the left'
- **Edge orientation**: Circular data interpreted $\mod \pi$

Edge filters

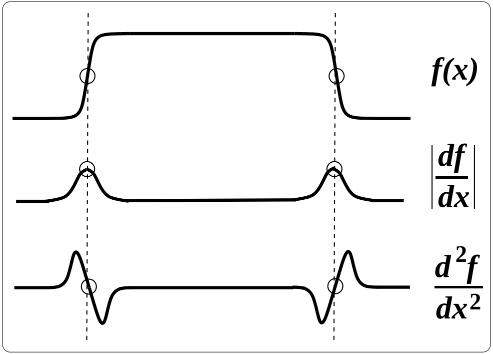
- Edge filters are high-pass filters using spatial derivatives of intensity function to
  - enhance intensity variation across the edge
  - suppress regions of constant intensity
- Operators applied in edge filtering
  - intensity **gradient** is **vector** composed of first order partial derivatives:

$$\nabla f(x, y) \doteq \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

  - **Laplace operator** is **scalar** composed of second order partial derivatives:

$$\Delta f(x, y) \doteq \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

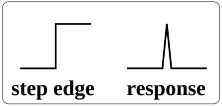
Signal and its first and second derivatives



- Edges are located at
  - maxima of absolute value of first derivative
  - **zero-crossings** of second derivative

Criteria for good edge filters

1. **No response to flat regions**
   $\Rightarrow$ sum of mask values is zero: $\sum_{r,c} w(r,c) = 0$
2. **Isotropy**
   - response is independent of edge orientation
3. **Good detection**: Minimise probabilities of
   - detecting spurious edges caused by noise (false positives)
   - missing real edges (false negatives)
4. **Good localisation**: Detected edges must be as close as possible to true edges
5. **Single response**: Minimise number of false local maxima around true edge



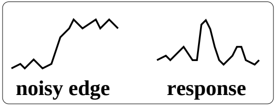
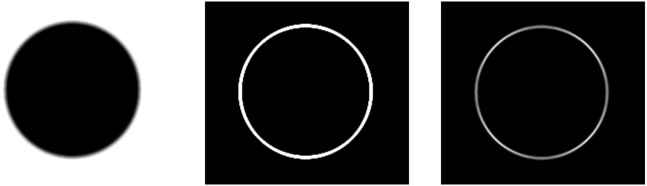


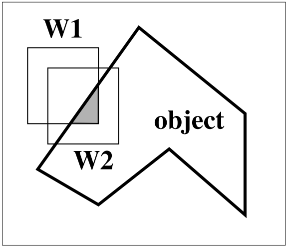
Illustration to isotropy criterion



original image | isotropic filter | anisotropic filter

- **Isotropic** filter: uniform edge magnitude for all directions
- **Anisotropic** edge filter: non-uniform magnitude
- In this illustration, response depends on edge orientation
  - directions $45^\circ \cdot k$ are amplified
  - directions $90^\circ \cdot k$ are suppressed

Illustration to single response criterion



- Same piece of contour detected in windows W1 and W2
  $\Rightarrow$ ‘phantom’ edges parallel to ‘true’ edges, thick contours
- Response depends on overlap between window and contour
- Multiple response typical for *all* window-based detection tasks

Gradient edge filters

Assume intensity function $f(x, y)$ is sufficiently smooth.

- Intensity **gradient** is **vector**

$$\nabla f(x, y) \doteq \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (f_x, f_y)$$

- **Magnitude** $M(x, y)$ and **orientation** $\Theta(x, y)$ of gradient are

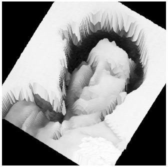
$$M(x, y) = \|\nabla f(x, y)\| = \sqrt{f_x^2 + f_y^2}$$

$$\Theta(x, y) = \arctan \frac{f_x}{f_y}$$

- Gradient vector gives direction and magnitude of *fastest growth of intensity*

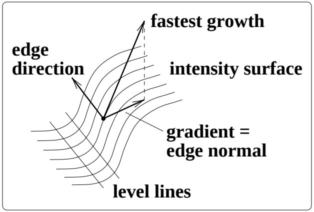
Meaning of gradient vector

original image



intensity surface

thresholded image



Intensity surface of edge and its gradient

Outline

3 × 3 gradient masks

- Partial derivatives are approximated by finite differences
  - ⇒ obtain gradient component masks $f * G_x = f_x$, $f * G_y = f_y$
  - ⇒ Y-masks are 90° rotations of X-masks
- Frequently used operators
  - Isotropic: less sensitive to edge orientation

Prewitt G_x

$$\frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Sobel G_x

$$\frac{1}{4}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Isotropic G_x

$$\frac{1}{2+\sqrt{2}}\begin{bmatrix} -1 & 0 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ -1 & 0 & 1 \end{bmatrix}$$

Outline

Properties of Canny edge detector

- Canny edge detector is **optimal** for noisy step edge if
  - image noise is additive, uncorrelated and Gaussian
  - edge filter is linear
- Optimality criterion combines
  - **good detection**
  - **good localisation**
- To satisfy **single response** criterion, two post-processing operations are used
  - *non-maxima suppression*
  - *hysteresis thresholding*
- Original Canny filter is quite complicated
  - more simple approximation is often use

Practical approximation of Canny filter

- Original Canny filter is quite complicated
- Simple practical approximation
  - apply **Gaussian filter** to smooth image: $g(x, y) = f(x, y) * w_G(x, y; \sigma)$

  $\Rightarrow$ parameter σ determines *size of filter*
  - apply **gradient operator** $\nabla g(x, y)$ to obtain edge magnitude and orientation
- Scale parameter σ is selected based on
  - desired level of detail: fine edges, global edges
  - noise level
  - localisation-detection trade-off

  $\Rightarrow$ see template matching

Fast implementation of Canny filter

- Use *associativity* of linear filters

$$\nabla\left(f(x,y) * w_G(x,y)\right) = f(x,y) * \left(\nabla w_G(x,y)\right)$$

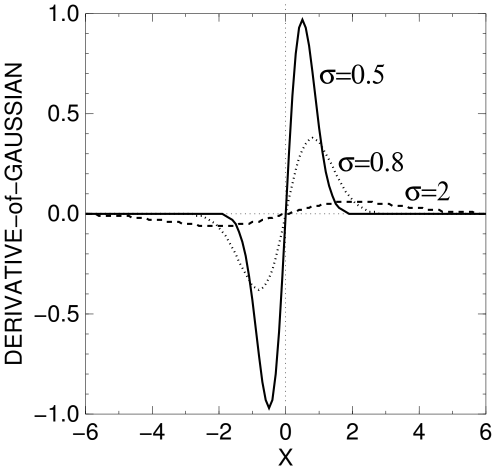
- Use **separability** of Gaussian $w_G(x,y) = w_G(x) \cdot w_G(y)$
- Obtain resulting vector filter (C is normaliser)

$$\nabla w_G(x,y) = \left(w_G(y) \cdot w'_G(x), w_G(x) \cdot w'_G(y)\right)$$

$$w'_G(x) \doteq \frac{\partial w_G(x)}{\partial x} = C \cdot x \exp\left\{-\frac{x^2}{2\sigma^2}\right\}$$

$\Rightarrow$ Filter is implemented as sequence of **1D masks**

Shape of $-w'_G(x)$ for growing σ



Outline

Edge localisation

- **Input**
  - edge magnitude (strength) $M(x, y)$
  - edge orientation $\Theta(x, y)$
- **Output**
  - binary *edge map*
  - $\Rightarrow$ 1 indicates edge, 0 no edge
- Selects maxima of $M(x, y)$ that are true edge pixels
- Usable with any filter that gives magnitude and orientation
  - gradient: Canny, Prewitt
  - non-gradient: Mérő & Vassy
- Includes two basic operations
  - **non-maxima suppression** to remove 'phantom' edges (considered)
  - **hysteresis thresholding** to remove noisy maxima (not considered)

Non-maxima suppression

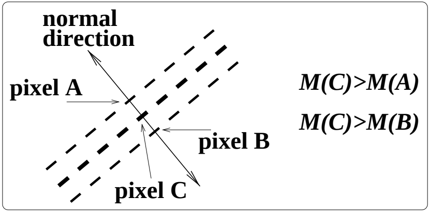
- Due to multiple response, edge magnitude $M(x, y)$ may contain wide ridges around local maxima
- Non-maxima suppression removes non-maxima pixels **preserving connectivity** of contours

Algoritmus: Non-maxima suppression

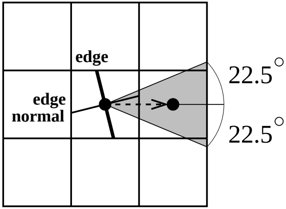
1. From each position (x, y), step in the two directions **perpendicular** to edge orientation $\Theta(x, y)$
2. Denote inital pixel (x, y) by C, the two neighbouring pixels in perpendicular directions by A and B
3. If $M(A) > M(C)$ or $M(B) > M(C)$, discard pixel (x, y) by setting $M(x, y) = 0$

Illustrations of non-maxima suppression

Point deletion



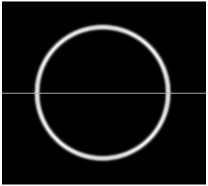
Step across edge



- *C* is not deleted since $M(C) > M(A)$, $M(C) > M(B)$
- *A*, *B* are deleted since $M(A) < M(C)$, $M(B) < M(C)$

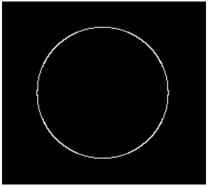
Select right and left neighbours of central pixel if angle of edge normal is in indicated range.

Example of non-maxima suppression



edge magnitude

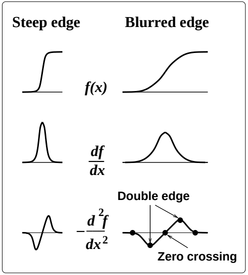
intensity profile along line



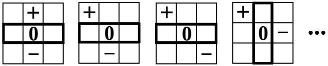
result of NMS

- Non-maxima suppression thins wide contour in edge magnitude image
- Intensity profile along indicated line is shown resized for better visibility

Principles of zero-crossing edge detector



principle of zero-crossing



masks for zero-crossings

- Laplace filter is applied to input image
- Zero-crossings are detected in filtered image
  - use simple masks *or*
  - local linear approximation and analytic solution

Convolution mask of LoG operator

- Use Laplacian-of-Gaussian (LoG)
  - smooth before taking derivatives
- In polar coordinates r, θ (origin in mask centre)

$$\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$$

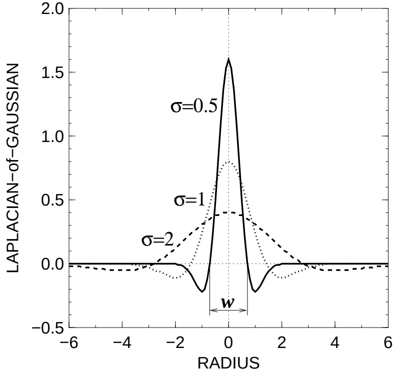
- Use associativity of linear filters and rotation symmetry of Gaussian to obtain (C is normaliser)

$$w_{zc}(r) = C\left(\frac{r^2}{\sigma^2} - 1\right)\exp\left\{\frac{-r^2}{2\sigma^2}\right\}$$

- Discrete zero-crossing mask
  - σ is scale parameter: small σ gives fine edges
  - cut LoG at $k\sigma$ similar to Gaussian

$\Rightarrow$ when $\sigma = 4$, size of mask is 40

Shape of LoG for varying σ



- 'Mexican hat'
- σ is scale parameter: level of odetail
  - smaller $\sigma \Rightarrow$ finer edges
- LoG filter is cut at $k\sigma$ similarly to Gauss filter.
- Size of positive central part: $w = 2\sqrt{2}\sigma$.

DoG filter: Difference-of-Gaussians

- LoG filter is not separable
  $\Rightarrow$ slow for large σ
- DoG filter is fast separable **approximation** of LoG filter

$$w_{LoG}(r;\sigma) \approx w_G(r;\sigma_1) - w_G(r;\sigma_2) \doteq w_{DoG}(r;\sigma_1,\sigma_2)$$

  - $w_G(r;\sigma_1)$, $w_G(r;\sigma_2)$ two Gauss filters
  - in general case, $\sigma \neq \sigma_1 < \sigma_2$
- Often used setting is $\sigma_1 = \sigma$, $\sigma_2 = 1.6\sigma$:

$$w_{DoG}(r;\sigma) = w_G(r;\sigma) - w_G(r;1.6\sigma)$$

Properties of zero crossing edge detector

- Continuous zero-crossing edge detector always gives **closed contours**
  - ⇒ continuous surface intersected by plane
  - in principle, this may help in contour following
  - in practice, many **spurious loops** appear
- Controlled operator size σ
  - natural edge hierarchy within *scale-space*
  - edges may only merge or disappear at rougher scales
  - ⇒ tree-like structure facilitates **structural analysis** of image
- Does not provide **edge orientation**
  - non-maxima suppression and hysteresis threshoding not applicable
  - another post-processing can be used to remove undesirable edges

Examples of edge detection by 15 $\times$ 15 LoG

LoG absolute

Post-processed

No post-processing

- **LoG absolute**: absolute value of output
  - dark lines are contours
- **Post-processed**: with removal of weak edges
- **No post-processing**: without removal of weak edges

Outline

Comparison of different edge detectors

Prewitt 3×3

Mérő-Vassy 7×7

LoG 21×21

Canny 3×3

Canny 7×7

Canny 25×25

- LoG result: with removal of weak edges
- Mérő-Vassy: non-gradient edge detector

Summary of edge detection

- 3×3 gradient operators (Prewitt, Sobel) are **simple and fast.** Used when
  - fine edges are only needed
  - noise level is low
- By varying σ parameter, **Canny operator** can be used
  - to detect fine as well as rough edges
  - at different noise levels
- All **gradient operators**
  - provide edge orientation
  - need localisation: non-maxima suppression, hysteresis thresholding
- **Zero-crossing** edge detector
  - is supported by neurophysiological experiments
  - was popular in the 1980's
  - today, **less frequently used** in practice