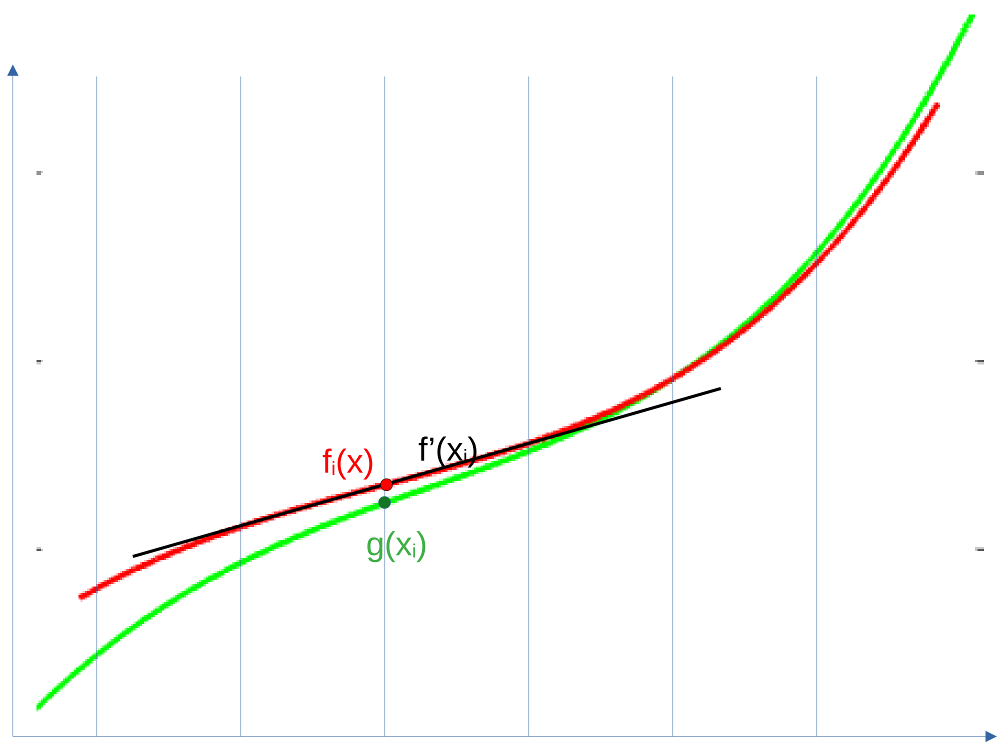
Simplified (1D) version of the Lucas-Kanade algorithm

May 6, 2025

1 Problem statement

The aim is to minimize the template matching score J, defined as

$$J = \sum_i \left(f(x_i, p_i + \Delta p_i) - g(x_i)\right)^2$$



2 Solution

For the first function $f(x)$, the first-order Taylor serie can be written as

$$f(x+\Delta x) = f(x) + f'(x)\Delta x$$

It is trivial that Δx depends on the parameter p. Therefore, it can be written that

$$x = h(p)$$

Then the first order Taylor serie can also be written to function $h(p)$

$$h(p+\Delta p) = h(p) + h'(p)\Delta p$$

In the sampling point, $x = h(p)$, therefore $\Delta x = h'(p)\Delta p$
If it is substituted to the original formula, then

$$f(x+\Delta x) = f(x) + f'(x)\Delta x = f(x) + f'(x)h'(p)\Delta p$$

Let us consider the score J again:

$$J = \sum_i \left(f(x_i, p_i + \Delta p_o) - g(x_i)\right)^2$$

$$J = \sum_i \left(f(x_i) + f'(x_i)h'(p_i)\Delta p - g(x_i)\right)^2$$

The optimal step, regarding the first order Taylor approximation, can be given by

$$\frac{\partial J}{\partial \Delta p} = 2\sum \left(f(x_i) + f'(x_i)h'(p_i)\Delta p - g(x_i)\right) f'(x_i)h'(p_i) = 0$$

The step can be calculated as

$$\sum \left[f'(x_i)h'(p_i)\right]^2 \Delta p = \sum \left(g(x_i) - f(x_i)\right) f'(x_i)h'(p_i)$$

Then

$$\Delta p = \frac{\sum \left(g(x_i) - f(x_i)\right) f'(x_i)h'(p_i)}{\sum \left[f'(x_i)h'(p_i)\right]^2}$$