

Object Detection

Tekla Toth

December 2021

Circle. Given $\mathbf{p}_i = [x_i \ y_i]^T$ in 2D, $i \in \{1 \dots N\}$, where \mathbf{p}_i the i th contour point of the object.

Circle parameters:

- $\mathbf{p}_0 = [x_0 \ y_0]^T$: Circle center
- r : Circle radius

Sphere. Given $\mathbf{p}_i = [x_i \ y_i \ z_i]^T$ in 3D, $i \in \{1 \dots N\}$, where \mathbf{p}_i the i th surface point of the object.

Sphere parameters:

- $\mathbf{p}_0 = [x_0 \ y_0 \ z_0]^T$: Sphere center
- r : Sphere radius

The problems can be formalized in a general description.

1 Minimal Fitting of Circle and Sphere

No. of input points:

- Circle: 3 points
- Sphere: 4 points

The solution based on intersection of lines / planes containing the center of the object.

Every 2 sample point defines a point and a normal vector to find a line / plane to estimate the center point.

$$\mathbf{p}_0 = \frac{\langle \mathbf{n}_i, \mathbf{m}_i \rangle}{\mathbf{n}_i} \quad (1)$$

2 Least Squares Fitting of Circle and Sphere

Minimization problem:

$$E(\mathbf{p}_0, r) = \sum_{i=1}^N (\|\mathbf{p}_0 - \mathbf{p}_i\| - r)^2, \quad (2)$$

In optimal case, the partial derivative of function E along x_0 , y_0 , z_0 , and r is zero at the minimum. This leads to a fixpoint iteration method:

$$\mathbf{p}_0 = \frac{1}{N} \sum_{i=1}^N r \frac{\mathbf{p}_0 - \mathbf{p}_i}{\|\mathbf{p}_0 - \mathbf{p}_i\|} \quad r = \frac{1}{N} \sum_{i=1}^N \|\mathbf{p}_0 - \mathbf{p}_i\|. \quad (3)$$

Initial estimation of required!